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# Some comments on the life and work of Jerzy K. Baksalary (1944–2005) [1]

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Following some biographical comments on Jerzy K. Baksalary (1944–2005), this article continues with personal comments by Oskar Maria Baksalary, Tadeusz Caliński, R. William Farebrother, Jürgen Groß, Jan Hauke, Erkki Liski, Augustyn Markiewicz, Friedrich Pukelsheim, Tarmo Pukkila, Simo Puntanen, Tomasz Szulc, Yongge Tian, Götz Trenkler, Júlia Volaufová, Haruo Yanai, and Fuzhen Zhang, on the life and work of Jerzy K. Baksalary, and with a detailed list of his publications. Our article ends with a survey by Tadeusz Caliński on Jerzy Baksalary's work in block designs and a set of photographs of Jerzy Baksalary.

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## Introduction

Professor Jerzy K. Baksalary passed away in Poznań, Poland, on 8 March 2005. He was 60 years old. Although suffering, he remained active in his research work to the very end.

Jerzy Baksalary was born in Poznań on 25 June 1944. He was awarded a PhD degree in 1975 by the Adam Mickiewicz University, Poznań, for his dissertation [4][2] in linear statistical models written under the supervision of Tadeusz Caliński.

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[1] This is a revised version of a set of handouts prepared for the Special Memorial Session for Jerzy Baksalary organized by Oskar Maria Baksalary, Simo Puntanen, George P. H. Styan, and Götz Trenkler at the 14th International Workshop on Matrices and Statistics held on the Albany Campus of Massey University in Auckland, New Zealand, 29 March–1 April 2005 and for the Southern Ontario Matrices and Statistics Days: Dedicated to Jerzy K. Baksalary (1944–2005) held in Windsor, Ontario, Canada, 9–10 June 2005. Part 3 of this article is a slightly revised version of the article by Tadeusz Caliński which appeared in the booklet prepared for the "Session on the occasion of the 60th birthday of Jerzy K. Baksalary" held at the Mathematical Research & Conference Center, Polish Academy of Sciences, Będlewo, Poland, on 17 August 2004.

[2] References in square brackets refer to the list of publications by Jerzy K. Baksalary in Table 2.1.

For the years 1969–1988, Jerzy Baksalary worked at the Agricultural University of Poznań, where from 1975 to 1988 he was associated with the Department of Mathematical and Statistical Methods, completing his Habilitationsschrift (post-doctoral dissertation) [55] in 1984. He joined the academic community in Zielona Góra in 1988, first working in the Department of Mathematics of the Tadeusz Kotarbiński Pedagogical University and then in the Institute of Mathematics of the University of Zielona Góra after it was founded in 2001.

He was Rector of the Tadeusz Kotarbiński Pedagogical University in Zielona Góra from 1990 to 1996 and then Dean of its Faculty of Mathematics, Physics, and Technology from 1996 to 1999. For the 1989–1990 academic year, he was Professor of the Finnish Academy of Sciences in the Department of Mathematical Sciences of the University of Tampere in Tampere, Finland.

Jerzy Baksalary published extensively on matrix methods for statistics. He is the author or coauthor of more than 170 research publications in linear algebra and statistics, including 45 papers published in *Linear Algebra and its Applications* (LAA). The Third Special Issue on Linear Algebra and Statistics of LAA [117] was edited by Jerzy K. Baksalary and George P. H. Styan.

At the funeral service for Jerzy Baksalary held in Poznań on 15 March 2005, Tadeusz Caliński eulogized him (in Polish):

> "Let me express our feelings particularly on behalf of those who were close to you in the early years of your academic career, in the seventies and eighties of the past century, at the Agricultural University of Poznań. At that time you were for us an encouraging example of a person full of scientific ideas and willing to work hard. Your works in the theory and application of mathematical statistics and linear algebra drew us into the streams of worldwide scientific literature.
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> Your personality stimulated younger colleagues and students, for whom you soon became a master and promoter of their careers. Among our joint scientific results of those years, your achievements shine with a particular brilliance. Your contributions to the Poznań school of mathematical statistics and biometry are highly esteemed at present and will be acknowledged by future generations."

A Special Memorial Session for Jerzy Baksalary was organized by Oskar Maria Baksalary, Simo Puntanen, George P. H. Styan, and Götz Trenkler at the 14th International Workshop on Matrices and Statistics held on the Albany campus of Massey University in Auckland, New Zealand, 29 March–1 April 2005. For this Memorial Session a set of handouts was prepared which included a reprint of a booklet prepared for the "Session on the occasion of the 60th birthday of Jerzy K. Baksalary" held at the Mathematical Research & Conference Center, Polish Academy of Sciences, Będlewo, Poland, on 17 August 2004, just before the 13th International Workshop on Matrices and Statistics. For the Auckland Memorial Session, Oskar Baksalary wrote about his father:

> "Although from the formal point of view I am a physicist and not a mathematician or statistician, with the death of JKB I have lost not only my father, but also my scientific master. On the one hand, this makes his passing away twice as hard for me to bear, but on the other hand I am very happy that for about the last four years I have been sharing with my father his great passion – mathematics.
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> During this period we have been spending lots of time together, for instance travelling, visiting jazz clubs and art galleries, attending Thursday seminars on linear algebra organized at the Agricultural University of Poznań, chatting, and first of all . . . doing mathematics.
>
> JKB really loved his subject and especially he was in love with everything having to do with matrices. This means he also loved the Workshops on Matrices and Statistics. My father and I have been participating in these Workshops since 2000, when the Workshop was held in Hyderabad, India, and thus the one organized this year in Auckland was to be the sixth which we would jointly attend."

The set of handouts distributed at the Memorial Session for Jerzy K. Baksalary in Auckland was revised and updated into a single 24-page handout for the Southern Ontario Matrices and Statistics Days: Dedicated to Jerzy K. Baksalary (1944–2005) held in Windsor, Ontario, Canada, 9–10 June 2005. This article is a further revision of the Windsor handout.

Apart from this introduction, the present article is in three parts. In Part 1 we present further personal comments on the life and work of Jerzy K. Baksalary by R. William Farebrother, Jürgen Groß, Jan Hauke, Erkki Liski, Augustyn Markiewicz, Friedrich Pukelsheim, Tarmo Pukkila, Simo Puntanen, Tomasz Szulc, Yongge Tian, Götz Trenkler, Júlia Volaufová, Haruo Yanai, and Fuzhen Zhang.

In Part 2 we discuss in detail the publications of Jerzy Baksalary, and in Table 2.1 below we present an annotated list which we believe to be complete of Jerzy Baksalary's publications in research journals and collections (conference proceedings, Festschriften, and other edited books), proposed problems and solutions to problems, and journal special issues, including references to reviews of his papers in *Mathematical Reviews* (MR) and *Zentralblatt MATH* (Zbl); for signed reviews the reviewer's name is given in parentheses.

Our article ends with Part 3 which is a survey by Tadeusz Caliński on Jerzy Baksalary's work in block designs, reprinted (with some minor changes) from the booklet prepared for the "Session on the occasion of the 60th birthday of Jerzy K. Baksalary" (Będlewo, Poland, August 2004).

## 1 Personal comments on the life and work of Jerzy K. Baksalary

I have known Jerzy Baksalary in various guises for more than thirty years. In the 1970s and early 1980s I received a steady stream of postcards from him requesting copies of my published and unpublished papers.

Unfortunately, Jerzy was not able to attend the 1983 Tampere Seminar on Linear Statistical Models. Thus I met him for the first time at a Multivariate Statistics Conference in Łódź (Poland) in 1986. At the Voorburg Workshop on Matrices and Statistics in 2001, Jerzy reminded me that Heinz Neudecker had reprimanded me in Tampere for not speaking proper 'Continental English' which has a different stress pattern from ordinary Received Pronounciation (e.g., *CE: ana-lȳsis* rather than *RP: a-naly̆sis*).

Jerzy and I met again at the 1987 Tampere Meeting and I recollect having prompted him to express the difficulty that Eastern Europeans then experienced in obtaining academic books and journals. The situation is only gradually being remedied following the accession of Poland and other Eastern European countries to the European Union. [As a continuing tribute to Jerzy's memory, may I urge anyone thinking of disposing of their surplus academic books and journals to send them to any of the numerous universities around the world that are still in urgent need of such donations.]

In Jerzy's review in *Mathematical Reviews* [MR567938 (82e:62097)] of my paper entitled "Estimation with aggregated data" [*Journal of Econometrics* 10, 43–55 (1979)] and in a subsequent paper [45] of his, Jerzy pointed out that the procedures I employed are formally invariant to the choice of a grouping matrix so that the distinct numerical results associated with the various choices of a generalised inverse are due to the presence of rounding errors. But for the fact that I had already done so, this observation may have prompted me to move on to other areas of research.

I do not recollect having cited Jerzy's work in any of my research papers, but in [142] he certainly helped me in generalising the solution to my problem entitled 'A class of square roots of involutory matrices' which I had proposed in *Image* [Problem 27.1 (2001), page 36] from the set of real nonsingular matrices to the set of nonzero complex matrices with group inverses.

Despite the fact that my principal fields of interest were distinct from his own, I have always found Jerzy to be very kind and considerate. What proved to be our final farewell after the Dortmund Workshop in 2003 was particularly touching.

R. William FAREBROTHER, *University of Manchester*

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I came across the papers of Jerzy K. Baksalary written together with numerous coauthors when I was working on my PhD thesis on mixed linear models, trying to adapt approaches in linear estimation to the estimation of fixed and random effects. Being myself inclined to linear algebra and matrix theory, I was intrigued by the statistical concepts such as linear sufficiency, linear admissibility, or minimum biased estimation, and their connection with linear matrix algebra.

The papers by him that I read had a clarity and aesthetic appeal in both the presentation and the way proofs were carried out which I had not encountered before. Therefore I tried to learn as much as possible, still today admiring the unrivalled ingenuity of the “Baksalarian way of thinking”. Only later did I discover numerous papers by Jerzy and his coauthors concerned with topics more in linear algebra than statistics, which then strongly influenced and stimulated the direction of my own research. Since Jerzy Baksalary has restarted to publish papers in recent years, I was eager to open a new file containing a collection of these. It is very sad, indeed tragic, that this file must now be closed so soon just when it seemed that a lot more fruitful research was to be expected.

Jürgen GROß, *Universität Dortmund*

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In 1976 when I started to work at the Department of Mathematical and Statistical Methods of the Agricultural University of Poznań there was a group of people there who were highly active scientists. Jerzy stood out among them for the clarity of presentation of his results and the precision of his questions at regularly held seminars. In 1978 Jerzy accepted me as a member of his team of collaborators and two years later I coauthored with Jerzy (and Radosław Kala) an article [32] published in 1980.

During the first Solidarity period (1980–1981) Jerzy was engrossed in the union’s activity at the Agricultural University of Poznań, and even outside it. Still, he was able to find time for scientific work. After the imposition of martial law, Jerzy was harassed by the secret police. This delayed the publication of his Habilitationsschrift (post-doctoral dissertation) in *Mathematische Operationsforschung und Statistik, Series Statistics* [55] (published in former East Germany) and hence he could not be the supervisor of my doctoral thesis, whose postulates were the results I had obtained in cooperation with him. Radosław Kala, his principal coauthor, was chosen to act as supervisor. This was no obstacle to my further cooperation with Jerzy and our next joint papers [56, 69, 89, 122] were published, respectively, in 1984, 1987, 1990 and 1994.

In the 1990s Jerzy was involved in administrative work at the Tadeusz Kotarbiński Pedagogical University in Zielona Góra (serving two terms as Rector and one as Dean), which forced a break in our cooperation. We started to work together again in 2002, and the effect was another three papers [143, 144, 156]. Two projects have been left unfinished because of his death. Work with Jerzy, consisting of hours of scientific discussion interwoven with discussion of the political and economic changes occurring in Poland, will always be one of my fondest memories of those years.

Jan HAUKE, *Adam Mickiewicz University, Poznań*

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The first time I met Jerzy in person was in 1984 when I attended, with Simo Puntanen, the *International Conference on Linear Statistical Inference* in Poznań. The party organized by Jan Hauke was memorable and Jerzy was a leading figure there. For example, discussions on Poland’s political situation were very fascinating. Jerzy was an activist in the free non-communist Solidarity labour union and had taken part in many actions. He was kept under surveillance by the state security service and sometimes detained. His vivid descriptions and his strong personal opinions about these historical events were unforgettable. I also remember his personal way of proposing a toast to empty sets. I feel that the four papers [15, 53, 60, 66] by Jerzy have influenced my research work the most.

Erkki LISKI, *University of Tampere*

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I first met Jerzy K. Baksalary in the late 1970s when I was looking for a topic for my M. Sc. thesis. Jerzy proposed a topic and helped me substantially with my work on it. I could come to his office at any time, ask him questions and discuss any research problem we were working on. I was especially welcome with solutions and new results. In such a case, he would postpone everything else to ask me for details and to study my results very carefully and most critically. Many of my conjectures were quickly rejected by him: he was a real master at constructing counterexamples! At about this time I started to attend a seminar on linear models guided by Jerzy and Radosław Kala. Jerzy's clear and precise talks accompanied with beautiful handwritten and well-organized presentations on the blackboard allowed us all to follow his lectures easily. His very good knowledge of almost all possible papers on his research area was extremely helpful in our discussions. He used to give from memory precise references to cited results. I had the opportunity to continue my research with Jerzy as the adviser for my PhD dissertation, publishing the results in six joint papers [60, 66, 77, 82, 90, 125] and [115, 124].

Our collaboration stopped in 1990 when Jerzy moved to Zielona Góra. A few years ago he started to attend a seminar on linear algebra and its applications guided by Tomasz Szulc and me in the Department of Mathematical and Statistical Methods of the Agricultural University of Poznań. His participation was warmly welcomed by everybody but especially by our PhD students, who were impressed by his talks, activity in discussion, presentation of open problems (often from *Image*) and proposals for their solutions. We all learnt a lot from our common work on these problems. A result of one such meeting is my joint paper [173] with Jerzy and his PhD student Paulina Kik.

After 1990, I continued my research on admissible estimation in the linear model, the main topic of my MSc thesis and PhD dissertation. This research was based on my papers with Jerzy as well as his papers: [38] and [65] with Radosław Kala, [67] and [78] with Thomas Mathew, and [106] with Götz Trenkler. Also another direction of my research - matrix orderings - was inspired by Jerzy's work and based on his papers: [84] with Friedrich Pukelsheim and George P. H. Styan, [102] with Sujit Kumar Mitra, [103] with Friedrich Pukelsheim, [122] with Jan Hauke and George P. H. Styan, and many others. Studying experimental design theory with my PhD students, I used to recommend the paper [55] to them; in [55] Jerzy compared, in a very nice and clear way, a linear model with its augmentation by nuisance parameters.

Augustyn MARKIEWICZ, *Agricultural University of Poznań*

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It is with great sadness that I have learnt of Jerzy's death. I have had the great privilege of being a quadruple coauthor of his [62, 84, 103, 111]. Our cooperation was started in 1985 by what I later came to view as a typical characteristic of Jerzy: the quest for mathematical completeness and elegance. During the 1984 Poznań Conference I had presented a joint paper with my colleague Karin Christof, who was my student then, presenting a sufficient condition for two matrices originating in the design of experiments to be Löwner comparable. Jerzy instantly asked whether our condition was also necessary. This gave rise to our first joint publication [62] in 1985, which did sharpen the condition to become indeed necessary and sufficient.

Jerzy insisted on a meticulous line-up of arguments to provide not just some answer to a question asked, but a complete answer which, in addition, could claim the maximum possible degree of mathematical elegance, and this prevailed throughout our further collaboration.

I'll certainly remember Jerzy as one of the Kings of Matrices that I had the pleasure to count among my coauthors.

Friedrich PUKELSHEIM, *Universität Augsburg*

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Science is the area of human life that should show and open new avenues for the future. Therefore science also needs pioneers who have a vision on the future and who have the capability and energy to open new paths. Professor Jerzy K. Baksalary was such a person. His scientific contributions are great.

Professor Baksalary, besides being a great scientist, was also an administrator. He served several years as the university rector. After the rectorship he returned to his scientific career, which evidently is not so common in the scientific community. This describes Jerzy K. Baksalary's versatile mental capabilities.

As Rector of the University of Tampere, I had the pleasure and great honour to have Jerzy K. Baksalary as a visiting professor. During the year he spent in our Department of Mathematical Sciences he wrote some 40 articles later published in top journals. This is a convincing indication on his scientific productivity.

Jerzy K. Baksalary was very interested in social questions and especially in the events which had important consequences for Poland during the last decades. I am convinced that Jerzy K. Baksalary was happy to live in the middle of the events which have also had an impact worldwide and which have led to deep changes in the Polish society.

Tarmo PUKKILA, *Ministry of Social Affairs and Health, Helsinki*
*Former Professor and Rector of the University of Tampere*

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Jerzy was a unique person and he was one of the most important persons in my academic career. I am a keen photographer, and I value having a substantial collection of photos of Jerzy. At the same time I also have a remarkable set of memories of events that involve him in a very colourful way.

In 1981 Erkki Liski and I jointly wrote a letter to Jerzy (and to his coauthor Radosław Kala) expressing our interest in their research. Indeed Jerzy's research interest and his style of writing papers was so surprisingly parallel to mine that I would often enthusiastically photocopy a paper of his whenever I saw one ... and I saw many of them!

Jerzy was invited to the Tampere Seminar on Linear Statistical Models in 1983 but, unfortunately, he could not come. The first time I met him face to face was in Poznań, 1984. In particular, Erkki and I enjoyed enormously a party held at Jan Hauke's home. Since then, I have had many good laughs with Jerzy and written several Baksalarian-style joint papers with him. The important factors between Jerzy and me over the years: research and sense of humour. One of the highlights was meeting him again at the Workshop on Matrices and Statistics in Hyderabad, India, December 2000, after a ten years' break (Jerzy's rectorship period): hugging took place immediately once we recognised each other, and there was Oskar also 10 years older!

Our society has lost a unique person but will not forget him.

Simo PUNTANEN, *University of Tampere*

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I was deeply saddened to learn that Jerzy has passed away. For the very last time we saw each other in mid-February at the seminar held at the Agricultural University of Poznań. This was a consecutive meeting in a series of seminars organized since 1999 every second Thursday. Participants of these seminars were: Jerzy K. Baksalary, Oskar Maria Baksalary, Jan Hauke, Augustyn Markiewicz, Tomasz Szulc and a group of PhD students – our group was called by Jerzy: PLAG, which is an acronym for the Poznań Linear Algebra Group. Our meetings were instructive and fruitful, and without any doubt, this was mainly due to Jerzy.

The activities of PLAG are well reflected in the problems and solutions in subsequent *Image* Problem Corners, for the problems proposed therein were extensively discussed and analyzed during our seminars. The fruit of the cooperation between Jerzy and myself within the PLAG meetings resulted in two joint papers [141, 170] published in *Linear Algebra and its Applications*.

Jerzy Baksalary was a referee of my habilitation thesis. He was known to be demanding and I was pleased that my scientific achievements were appreciated by him. With the death of Jerzy Baksalary, the linear algebra community has lost a truly great specialist in matrix analysis and PLAG has lost its leader.

Tomasz SZULC, *Adam Mickiewicz University, Poznań*

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Some of Baksalary's work is concerned with solving linear matrix equations using generalized inverses of matrices; this work started in the late 1970s. Using generalized inverses, Penrose ["A generalized inverse for matrices", *Proceedings of the Cambridge Philosophical Society* 51, 406–413 (1955)] had shown that the matrix equation $AX = B$ is solvable for $X$ iff $AA^{-}B = B$, and then the general solution can be written as

$$X = A^{-}B + (I - A^{-}A)U,$$

where $U$ is arbitrary; similarly $AXB = C$ is solvable iff $AA^{-}CB^{-}B = C$, and in this case the general solution can be written as

$$X = A^{-}CB^{-} + (I - A^{-}A)U_1 + U_2(I - BB^{-}),$$

where $U_1$ and $U_2$ are arbitrary. These two results give the key applications of generalized inverses for solving linear matrix equations.

Great difficulty is encountered, however, in solving some more general linear matrix equations. Jerzy K. Baksalary and Radosław Kala were two pioneers in solving the two matrix equations $AX - YB = C$ and $AXB + CYD = E$ using generalized inverses. In two papers [23, 38] by Baksalary and Kala published in 1979 and 1980 and in the paper [43] by Baksalary in 1982, it was established that $AX + YB = C$ is consistent iff

$$(I - AA^{-})C(I - B^{-}B) = 0,$$

and in this case the general solution is

$$X = AC^{-} + A^{-}Z + A^{-}ZB + (I - AA^{-})W,$$

$$Y = -(I - AA^{-})C^{-}B + A^{-}Z - (I - AA^{-})ZBB^{-},$$

where $W$ and $Z$ are arbitrary. This result shows that the solvability condition and the general solution of the equation can be expressed by generalized inverses.

Moreover, Baksalary and Kala [38] gave the solvability condition and the general solution of the equation $AXC + CYD = E$ by generalized inverses. This inspired a variety of subsequent works in the 1990s and 2000s on $AX + YB = C$ and $AXC + CYD = E$, for example, properties of their solutions, least-squares solutions of the two equations, minimal ranks of $AX + YB - C$ and $AXC + CYD - E$, etc.

Yongge TIAN, *Shanghai University of Finance and Economics*

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I first came across Jerzy's name when I attended the International Tampere Seminar on Linear Statistical Models and their Applications in Tampere, Finland, in 1983. Several people from the Eastern European countries had also been invited by the organizers but almost none of them showed up. Jerzy Baksalary and Radek Kala from the Agricultural University of Poznań were not allowed to come to Tampere. After all, we were at the height of the Cold War then.

In the 1980s I had become interested in the performance of restricted least squares and pre-test estimators, which are of some importance in econometrics. One of my favourite topics was the "Comparison of Least Squares and Restriced Least Squares Estimators". Consider the linear regression model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$ where $\mathbf{X}$ is of full column rank, $E(\mathbf{u}) = \mathbf{0}$ and $Cov(\mathbf{u}) = \sigma^2\mathbf{I}$. Then the least squares estimator (LSE) of $\boldsymbol{\beta}$ is $\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$. Suppose we have additional linear restrictions on the parameter vector $\beta$ in the form $\mathbf{R}\boldsymbol{\beta} = \mathbf{r}$, where $\mathbf{R}$ is of full row rank. The corresponding Restricted Least Squares Estimator (RLSE) is

$$\mathbf{b} = \hat{\boldsymbol{\beta}} - (\mathbf{X}'\mathbf{X})^{-1}\mathbf{R}'[\mathbf{R}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{R}']^{-1}(\mathbf{R}\hat{\boldsymbol{\beta}} - \mathbf{r}).$$

It is well known that

$$\mathrm{cov}(\hat{\boldsymbol{\beta}}) = \sigma^2(\mathbf{X}'\mathbf{X})^{-1}, \quad \mathrm{cov}(\mathbf{b}) = \sigma^2[(\mathbf{X}'\mathbf{X})^{-1} - \mathbf{G}],$$

where

$$\mathbf{G} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{R}'[\mathbf{R}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{R}']^{-1}\mathbf{R}(\mathbf{X}'\mathbf{X})^{-1}.$$

If the condition $\mathbf{R}\boldsymbol{\beta} = \mathbf{r}$ is violated, the RLSE($\mathbf{b}$) becomes biased. Nevertheless there is some potential in $\mathbf{b}$ to outperform the LSE $\hat{\boldsymbol{\beta}}$ with regard to the matrix mean square error (MMSE) criterion. Actually the difference of the MMSE matrices is given by

$$\mathbf{M}(\hat{\boldsymbol{\beta}}) - \mathbf{M}(\mathbf{b}) = \sigma^2\mathbf{G} - \mathbf{H}\boldsymbol{\delta}\boldsymbol{\delta}'\mathbf{H}',$$

where

$$\mathbf{H} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{R}'[\mathbf{R}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{R}']^{-1} \quad \text{and} \quad \boldsymbol{\delta} = \mathbf{R}\boldsymbol{\beta} - \mathbf{r}$$

Consulting a well-known result from matrix theory, we see that $\mathbf{M}(\hat{\boldsymbol{\beta}}) - \mathbf{M}(\mathbf{b}) \geq_L \mathbf{0}$, i.e. $\mathbf{b}$ is better than $\hat{\mathbf{b}}$ with respect to the MMSE criterion if and only if

$$\boldsymbol{\delta}'\mathbf{H}'\mathbf{G}^{-1}\mathbf{H}\boldsymbol{\delta} \leq \sigma^2.$$

A similar result had been achieved by Toro-Viczarrondo & Wallace [*Journal of the American Statistical Association* 63, 558–572 (1968)], but I was happy to derive the preceding equivalence without using their tedious derivations involving eigenvalues. Using this approach, I was able to obtain a number of additional theorems, relying heavily on the matrix $\mathbf{G}^{-1}$. The manuscript was soon ready for submission, but one day before sending it off to the Editor of *Communications in Statistics–Theory and Methods*, I realized that the matrix $\mathbf{G}$ was *singular*. The paper disappeared into a dark corner of my study to slumber there for two years.

In 1983, by chance I browsed through some not so well-known journal in our library entitled *Bulletin of the Polish Academy of Sciences* and found the article [48] by Baksalary and Kala under the title: “Partial orderings between matrices one of which is of rank one”. Its main result saved my paper:

THEOREM: *Let* $\mathbf{A} \in \mathbb{C}^{n\times n}$ *be Hermitian,* $\mathbf{a} \in \mathbb{C}^n$ *and* $\alpha > 0$*. Then* $\alpha\mathbf{A} - \mathbf{a}\mathbf{a}^* \geq_L \mathbf{0}$ *if and only if*

(i) $\mathbf{A} \geq_L \mathbf{0}$
(ii) $\mathbf{a} \in \mathcal{R}(\mathbf{A})$
(iii) $\mathbf{a}^*\mathbf{A}^-\mathbf{a} \leq \alpha$,

*where* $\mathbf{A}^-$ *is any generalized inverse of* $\mathbf{A}$.

Using this important theorem I could revive and repair my manuscript. For example, the now correct criterion for dominance of $\hat{\mathbf{b}}$ over $\hat{\boldsymbol{\beta}}$ became

$$\boldsymbol{\delta}'[\mathbf{R}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{R}']^{-1}\boldsymbol{\delta} \leq \sigma^2.$$

My paper was published in *Communications in Statistics–Theory and Methods* 14, 2495–2509 (1985).

Some years later I received an invitation to participate in the International Conference on Mathematical Statistics in Kozubnik, Poland. The Solidarity movement had already been founded then with Lech Wałęsa as its leader (becoming later Poland's President). During the conference I had the opportunity to meet Jerzy for the first time.

I was immediately fascinated by Jerzy's style of presenting and explaining his research results. His transparencies were perfectly prepared in a remarkably beautiful handwriting. I liked his crystal-clear, but nevertheless high-level and original presentations from the very beginning. This was my first experience attending a conference in Poland. After the lectures and after some vodkas, the participants had a chance to get better acquainted with each other at the informal meeting during the evening. Normally, the discussions started with some mathematics and ended up with hot debates over politics.

It was embarrassing for me to notice that the Polish participants expressed their opinion about their government very freely, whereas my colleagues from East Germany would not do the same, obviously fearing that somebody might report them to the Secret Police when they returned to the German Democratic Republic, the official name of East Germany. Jerzy hated communism, and even more its supporters. Having become one of the front leaders of Solidarity in Poznań, he had to spend some days in prison.

About that time our first joint project started. I applied for a scholarship for Jerzy at the Alfried Krupp Foundation of Germany. Ironically, Jerzy's father had worked as a coal miner close to Dortmund after World War I in a pit owned by Alfried Krupp, one of Germany's richest men. Incidentally, Krupp had made a fortune by selling arms to the German Emperor, Kaiser Wilhelm. When Poland became independent, Jerzy's father went back to Poland.

Jerzy got the support and visited Dortmund for three months in 1988. We had a good time together. Jerzy worked very hard on matrices and statistics, and even on Sundays, after we had had lunch together in our house, he returned to the university to resume his thinking about new theorems. I was very impressed with his intellectual abilities. Equipped with a sharp mind, a photographic memory and a broad imagination, he was able to put forward and solve many problems. My estimate is that he finished over ten papers during his short stay in Dortmund. Occasionally, I suggested some problems to him, and often he came up with some neat counterexamples, mainly by presenting some matrices of low order.

When communism had lost its power in Poland, he became professor at the Tadeusz Kotarbiński Pedagogical University in Zielona Góra, a town close to the German border. In 1990, he took over the rectorship of this University, and, alas, stopped his research activities to devote all his strength to administrative tasks. Fortunately, he came back to science in 2000, and we resumed our collaboration. We wrote a joint paper (together with his son Oskar), and planned to investigate the determinant of modified matrices.

To give an impression of the extent of Jerzy's work, I have prepared the following list of topics he has worked on:

- Pre-test estimation
- Experimental design
- Structure of dispersion matrices
- Partial orderings (Löwner, star, minus)

- Estimation in regression under restrictions
- Invariance (rank, trace, eigenvalues, singular values, norms)
- Admissibility
- Variance components estimation
- Estimation in linear models under different dispersion matrix structures
- Equality of the OLSE and the BLUE
- Matrix equations and inequalities
- Aggregation of data
- Covariance adjustment technique
- Symmetrizes of matrices
- Prediction in linear models
- Estimability
- Growth curves
- Euclidian distance between estimators
- Estimation in the presence of nuisance parameters
- Canonical correlation
- Distribution of quadratic forms
- Oblique, orthogonal, generalized, hypergeneralized projectors
- Generalized inverses
- Modified matrices

On the 8th of March, Jerzy died. I am very sad, and I wish to express my deep respect for him by using one of his favourite expressions (in his own pronounciation)

”Jerzy, you were an unbillivable man”.

Götz TRENKLER, *Universität Dortmund*

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As every day, this morning (Monday, March 21, 2005) I opened my e-mail in my office and there it was. The announcement from ILAS-NET about the sudden death of Jerzy. It is one of those moments that hit you in your face very unexpectedly. There are people in the big world whom even if we do not know very closely in person, their names accompany us over many, many years in our professional life and somehow we are constantly aware of their existence even without special thinking. Jerzy, in my case, was one of them. In an instant moment memories brought back the time when the name Baksalary came up on an almost everyday basis in my work. If I step back in time and think about the period when I was working with Lubomír Kubáček in Bratislava and tried hard to make some progress with my PhD research, dealing a great deal with linear models, subjects like linear projectors in connection with estimability, admissibility, restrictions in linear models, singular models, or nuisance parameters troubled my mind a lot.

That was the time when I started to get familiar with Jerzy Baksalary's work. I remember reading very thoroughly and in detail many of his papers and finding answers in them to many of my questions. Here is a short list of some just as a sample of those that I thought were the most influential [19, 25, 26, 27, 40, 41] and then later [59] or [99].

Then came the time when I started to look more closely at topics regarding nuisance parameters in linear models and there were again at least several papers by Jerzy that I found extremely useful. The papers [55, 68, 99] were considered to contain important information when we were putting together the chapters on nuisance parameters in my joint monograph with Lubomír Kubáček and his late wife Ludmila Kubáčková [*Statistical Models with Linear Structures*, pub. Veda: Publishing House of the Slovak Academy of Sciences, Bratislava, 1995].

Then, several years later I became fortunate and for the first time I met Jerzy in person at the International Workshop on Matrices and Statistics in Hyderabad, India, December 2000. He was already not in good health but always smiling, very kind, a real gentleman and indeed, very productive and active. Again, here I just mention two of his papers that I looked through not too long ago. They are [145] and [155].

His insight and active work during that recent time was and still is a very reliable source of information for me. And, it will be for many more years. Jerzy will be never forgotten.

Júlia VOLAUFOVÁ
*Louisiana State University Health Sciences Center, New Orleans*

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Dear Oskar: It was a profound shock to hear of the sudden death of your great father Dr. Jerzy K. Baksalary. Your sorrow will be shared by everyone in the world who knew and loved him. I send my deepest sympathy to you.

As you know, I have a joint paper [114] with your father, and among the many papers written by your father, I was most influenced by the 46 equivalent conditions given in the invited paper [68] entitled "Algebraic characterizations and statistical implications of the commutativity of orthogonal projectors".

I was honoured that your father kindly included two conditions given in my joint paper with C. Radhakrishna Rao [*Journal of Statistical Planning and Inference* 3, 1–17 (1979)] among the 46 equivalent conditions in [68], and owing to this paper by your father I have been motivated to work more on projectors, both orthogonal and oblique.

Haruo YANAI
*The National Center for University Entrance Examination, Tokyo*

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Jerzy passed away: the linear algebra community lost an active researcher and we lost a friend. He was a nice man and very fine mathematician. I knew Jerzy from his papers long before I met him in person. His contributions to matrix theory particularly in the area of matrix orderings are of fundamental importance. I met Jerzy only a few times at meetings. He had many interests and a great enthusiasm for mathematics. He will be remembered!

Fuzhen ZHANG
*Nova Southeastern University, Fort Lauderdale, Florida*

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## 2 Publications by Jerzy K. Baksalary

In Table 2.1 below we present an annotated list, which we believe to be complete, of Jerzy Baksalary's publications in research journals and collections (conference proceedings, Festschriften, and other edited books), solutions to problems, and journal special issues, including references to reviews of his papers in *Mathematical Reviews* (MR) and *Zentralblatt MATH* (Zbl); for signed reviews the reviewer's name is given in parentheses. For reviews in *Mathematical Reviews*, the new style review number (six or seven digits) is given; for all reviews except for the very latest the old-style number is given in parentheses.

The $181$ entries in Table 2.1 are listed chronologically, and by authorship within year, and may be classified as follows:

$127$ research papers in $32$ peer-refereed research journals; $45$ papers in *Linear Algebra and its Applications* and $23$ in the *Journal of Statistical Planning and Inference*(see Table 2.2);

$12$ research papers in research collections (conference proceedings, Festschriften, and other edited books);

$31$ solutions to research problems ($28$ in *Image: The Bulletin of the International Linear Algebra Society*, and one each in *Econometric Theory*, *The IMS Bulletin*, and *Statistica Neerlandica*);

$11$ other items, being Jerzy Baksalary's PhD dissertation, two journal special issues (one of *Linear Algebra and its Applications* and one of the *Journal of Statistical Planning and Inference*), two research problems (both in *Image*), four papers submitted for publication in research journals, and two papers in preparation.

Associated with a bibliography by a particular author with several coauthors, we may define an "authorship matrix" $\mathbf{A} = \{a_{ij}\}$, where $a_{ij} = 1$ if bibliographic entry number $i$ is written with coauthor number $j$ and $a_{ij} = 0$ otherwise. The authorship matrix $\mathbf{A}$ for Jerzy Baksalary based on Table 2.1 is $181 \times 43$ and the diagonal entries of $\mathbf{A}'\mathbf{A}$ represent the numbers of bibliographic entries written with each of the $43$ coauthors and these numbers are presented in Table 2.3. In Table 2.4 are listed the $11$ coauthors with more than $3$ entries in Table 2.1.

We computed the eigenvalues of the $43 \times 43$ matrix $\mathbf{A}'\mathbf{A}$ and found that $11$ of these eigenvalues are greater than $3$. For these $11$ eigenvalues we computed the corresponding eigenvectors and identified the coauthor corresponding to the entry that is largest in absolute value. These $11$ eigenvalues and associated coauthors are given in Table 2.5. Interestingly, we note that the $11$ coauthors in Table 2.5 coincide with the $11$ coauthors in Table 2.4 but appear there in a different order than in Table 2.5. The first two coauthors and the last two coauthors in Tables 2.4 and 2.5, however, coincide and appear in the same order.

Jerzy Baksalary supervised four PhD dissertations, all at the Adam Mickiewicz University, Poznań (see Table 2.6) and he published 17 reviews in *Mathematical Reviews*, see Table 2.7; almost all of these reviews are extensive and many identify misprints in the article under review.

TABLE 2.1: Annotated complete list of publications by Jerzy K. Baksalary.

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[1] Jerzy Baksalary, Radosław Kala (1973). Wyznaczanie bazy macierzy. *Roczniki Akademii Rolniczej w Poznaniu: Algorytmy Biometryczne i Statystyczne* 64 (2), 3–9. [Part number refers to the series *Algorytmy Biometryczne i Statystyczne*. Article has # ABS-11 and is in Polish. English translation of article title: “The determination of a basis of a matrix”.]

[2] Jerzy Baksalary, Radosław Kala (1974). Metody analizy doświadczeń nieortogonalnych. In *Czwarte Colloquium Metodologiczne z Agro-Biometrii (Poznań, 10–15 września 1974)* (Eugeniusz Bilski, Tadeusz Caliński, Witold Klonecki, Wiktor Oktaba, eds.), Komitet Hodowli i Uprawy Roślin Polskiej Akademii Nauk i Polskie Towarzystwo Biometryczne, Warszawa, 1974, pp. 201–258. [Article in Polish. English translation of article title: “Methods for analysing nonorthogonal experiments”. English translation of collection title: *Fourth Colloquium on Methodology in Agricultural Biometry: Poznań, 10–15 September 1974*. English translation of publisher: “Polish Academy of Sciences and Polish Biometrical Society, Warsaw”.]

[3] Jerzy Baksalary, Radosław Kala (1974). Procedura obliczania uogólnionej odwrotności macierzy. *Roczniki Akademii Rolniczej w Poznaniu: Algorytmy Biometryczne i Statystyczne* 71 (3), 157–165. [Part number refers to the series *Algorytmy Biometryczne i Statystyczne*. Article has # ABS-30 and is in Polish. English translation of article title: “Procedure for calculating a generalized inverse of any matrix”.]

[4] Jerzy K. Baksalary (1975). Estymowalność funkcji parametrycznych w modelach liniowych. PhD dissertation, Adam Mickiewicz University, Poznań. [Dissertation in Polish. English translation of title: “Estimability of the parametric functions in linear models”.]

[5] Jerzy Baksalary, Anita Dobek, Radosław Kala (1975). Rozwiązywanie równań liniowych z nieujemnie określoną symetryczną macierzą układu. *Roczniki Akademii Rolniczej w Poznaniu: Algorytmy Biometryczne i Statystyczne* 80 (4), 243–260. [Part number refers to the series *Algorytmy Biometryczne i Statystyczne*. Article has # ABS-40 and is in Polish. English translation of article title: “Procedure for solving a nonnegative definite symmetric system of equations”.]

[6] Jerzy Baksalary, Anita Dobek, Radosław Kala (1976). Wyznaczanie operatorów rzutowania ortogonalnego. *Roczniki Akademii Rolniczej w Poznaniu: Algorytmy Biometryczne i Statystyczne* 86 (5), 187–194. [Part number refers to the series *Algorytmy Biometryczne i Statystyczne*. Article has # ABS-49 and is in Polish. English translation of article title: “Computation of the orthogonal projectors”.]

[7] J. K. Baksalary, A. Dobek, R. Kala (1976). A method for computing projectors. *Žurnal Vyčislitel′noĭ Matematiki i Matematičeskoĭ Fiziki* 16, 1038–1040. [Article in English (with abstract and two references in Russian); MR421063 (54 #9068, L. W. Ehrlich); Zbl 0338.65024. Reprinted (with English translation of abstract and two references by J. Berry) in *U.S.S.R. Computational Mathematics and Mathematical Physics* 16 (1976), 216–218 (1977). Zbl 0357.65029.]

[8] Jerzy K. Baksalary, Radosław Kala (1976). Extensions of Milliken’s estimability criterion. *The Annals of Statistics* 4, 639–641. [MR415900 (54 #3978, C. R. Rao); Zbl 0336.62058.]

[9] Jerzy K. Baksalary, Radosław Kala (1976). Criteria for estimability in multivariate linear models. *Mathematische Operationsforschung und Statistik* 7, 5–9. [MR413375 (54 #1489, C. G. Khatri); Zbl 0329.62053.]

[10] J. K. Baksalary, A. Dobek, R. Kala (1977). A method of finding bases of a matrix. *Annales Societatis Mathematicae Polonae, Series I: Commentationes Mathematicae* 20, 1–5. [MR453773 (56 #12027); Zbl 0364.65029.]

[11] Jerzy Baksalary, Anita Dobek, Radosław Kala (1977). Wyznaczanie operatorów rzutowania. *Roczniki Akademii Rolniczej w Poznaniu: Algorytmy Biometryczne i Statystyczne* 95 (6), 175–183. [Article has # ABS-60 and is in Polish. English translation of article title: "The computation of projectors".]

[12] Jerzy K. Baksalary, Radosław Kala (1977). An extension of a rank criterion for the least squares estimator to be the best linear unbiased estimator. *Journal of Statistical Planning and Inference* 1, 309–312. [MR518938 (58 #24745, George P. H. Styan); Zbl 0383.62041.]

[13] Jerzy K. Baksalary, Radosław Kala (1977). Sums of squares and products of matrices for a non-full ranks hypothesis in the model of Potthoff and Roy. *Mathematische Operationsforschung und Statistik, Series Statistics* 8, 459–465. [MR501598 (58 #18914, J. A. John); Zbl 0397.62035.]

[14] Jerzy Baksalary, Radosław Kala, Krystyna Katulska (1977). Analiza wariancji dla klasyfikacji krzyżowych metodą Bocka. *Roczniki Akademii Rolniczej w Poznaniu: Algorytmy Biometryczne i Statystyczne* 95 (6), 3–32. [Part number refers to the series *Algorytmy Biometryczne i Statystyczne*. Article has # ABS-51 and is in Polish. English translation of article title: "Analysis of variance for cross classifications by the method of Bock".]

[15] J. K. Baksalary, L. C. A. Corsten, R. Kala (1978). Reconciliation of two different views on estimation of growth curve parameters. *Biometrika* 65, 662–665. [MR521835 (80g:62034, C. G. Khatri); Zbl 0398.62063.]

[16] Jerzy Baksalary, Anita Dobek, Radosław Kala (1978). Analiza statystyczna w ogólnym modelu liniowym. *Roczniki Akademii Rolniczej w Poznaniu: Algorytmy Biometryczne i Statystyczne* 106 (7), 3–23. [Part number refers to the series *Algorytmy Biometryczne i Statystyczne*. Article has # ABS-61 and is in Polish. English translation of article title: "Statistical analysis in a general linear model".]

[17] Jerzy Baksalary, Anita Dobek, Radosław Kala (1978). Estymacja krzywych wzrostu. *Roczniki Akademii Rolniczej w Poznaniu: Algorytmy Biometryczne i Statystyczne* 106 (7), 81–113. [Part number refers to the series *Algorytmy Biometryczne i Statystyczne*. Article has # ABS-65 and is in Polish. English translation of article title: "Estimation of growth curves".]

[18] Jerzy Baksalary, Anita Dobek, Radosław Kala (1978). Rozkład macierzy rzeczywistej na czynniki pełnych rzędów. *Roczniki Akademii Rolniczej w Poznaniu: Algorytmy Biometryczne i Statystyczne* 106 (7), 179–183. [Part number refers to the series *Algorytmy Biometryczne i Statystyczne*. Article has # ABS-69 and is in Polish. English translation of article title: "Rank factorization of a real matrix".]

[19] Jerzy K. Baksalary, Radosław Kala (1978). A bound for the Euclidean norm of the difference between the least squares and the best linear unbiased estimators. *The Annals of Statistics* 6, 1390–1393. [MR523772 (80f:62065, Lubomír Kubáček); Zbl 0392.62051.]

[20] Jerzy K. Baksalary, Radosław Kala (1978). Estymowalność liniowych funkcji parametrycznych w jednowymiarowym modelu liniowym. *Roczniki Polskiego Towarzystwa Matematycznego, Seria III: Matematyka Stosowana* 12, 133–144. [MR517468 (80d:62046, R. Zmyślony); Zbl 0439.62045. Article in Polish. English translation of article title: "Estimability of linear parametic functions in a one-dimensional linear model".]

[21] Jerzy K. Baksalary, Radosław Kala (1978). Estymowalność liniowych funkcji parametrycznych w jednowymiarowym modelu liniowym z restrykcjami. *Roczniki Polskiego Towarzystwa Matematycznego, Seria III: Matematyka Stosowana* 12, 145–151. [MR517469 (80d:62047, R. Zmyślony); Zbl 0439.62046. Article in Polish. English translation of article title: "Estimability of linear parametric functions in a one-dimensional linear model with restrictions".]

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[23] J. K. Baksalary, R. Kala (1979). The matrix equation $\mathrm{AX} - \mathrm{YB} = \mathrm{C}$. *Linear Algebra and its Applications* 25, 41–43. [MR528711 (80d:15014, Harald K. Wimmer); Zbl 0403.15010.]

[24] Jerzy K. Baksalary, Radosław Kala (1979). Two relations between oblique and $\Lambda$-orthogonal projectors. *Linear Algebra and its Applications* 24, 99–103. [MR524829 (80c:15013, M. F. Smiley); Zbl 0401.15004.]

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[26] J. K. Baksalary, R. Kala (1979). On the prediction problem in the seemingly unrelated regression equations model. *Mathematische Operationsforschung und Statistik, Series Statistics* 10, 203–208. [MR544566 (81b:62118, Kenneth N. Berk); Zbl 0417.62053.]

[27] J. K. Baksalary, R. Kala (1979). Covariance adjustment when a vector of parameters is restricted to a given subspace. *SIAM Journal on Applied Mathematics* 37, 20–21. [MR536300 (80m:62063, Federico O'Reilly); Zbl 0411.62033.]

[28] Jerzy K. Baksalary, Tadeusz Caliński, Radosław Kala (1980). Estymacja krzywych wzrostu i jej zastosowanie do oceny odmian gatunków o plonowaniu wielokrotnym. *Biuletyn Oceny Odmian / Cultivar Testing Bulletin (Poznań)* 8, 167–181. [Article in Polish. English translation of article title: "Estimation of growth curves and its application in evaluating varieties of species with repeated yielding".]

[29] Jerzy K. Baksalary, Anita Dobek, Radosław Kala (1980). A necessary condition for balance of a block design. *Biometrical Journal / Biometrische Zeitschrift* 22, 47–50. [MR576961 (81h:62134, K. L. Sharma); Zbl 0457.62059.]

[30] Jerzy K. Baksalary, Anita Dobek, Radosław Kala (1980). Some methods for constructing efficiency-balanced block designs. *Journal of Statistical Planning and Inference* 4, 25–32. [MR587028 (81j:62149); Zbl 0461.62066.]

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[32] J. K. Baksalary, J. Hauke, R. Kala (1980). Nonnegative definite solutions to some matrix equations occurring in distribution theory of quadratic forms. *Sankhyā, The Indian Journal of Statistics: Series A* 42, 283–291. [MR656262 (83e:15015, John P. Daughtry); Zbl 0501.62036.]

[33] J. K. Baksalary, R. Kala (1980). A new bound for the Euclidean norm of the difference between the least squares and the best linear unbiased estimators. *The Annals of Statistics* 8, 679–681. [MR568730 (81c:62070, Simon Dahan); Zbl 0464.62055.]

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[38] J. K. Baksalary, R. Kala (1980). The matrix equation $\mathrm{AXB} + \mathrm{CYD} = \mathrm{E}$. *Linear Algebra and its Applications* 30, 141–147. [MR568786 (82e:15010); Zbl 0437.15005.]

[39] J. K. Baksalary, R. Kala (1980). On the difference between two second degree polynomials, each following a chi-square distribution. *Sankhyā, The Indian Journal of Statistics: Series A* 42, 123–127. [MR637997 (82k:62029); Zbl 0487.62043.]

[40] J. K. Baksalary, R. Kala (1981). Linear transformations preserving best linear unbiased estimators in a general Gauss–Markoff model. *The Annals of Statistics* 9, 913–916. [MR619297 (82h:62100, Yasuko Chikuse); Zbl 0471.62067.]

[41] J. K. Baksalary, R. Kala (1981). Simple least squares estimation versus best linear unbiased prediction. *Journal of Statistical Planning and Inference* 5, 147–151. [MR627238 (82k:62122); Zbl 0476.62057.]

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[44] Jerzy K. Baksalary, Radosław Kala (1982). Admissible estimation by covariance adjustment technique. *Sankhyā, The Indian Journal of Statistics: Series A* 44, 281–285. [MR688807 (84c:62016).]

[45] Jerzy K. Baksalary (1983). An invariance property of Farebrother's procedure for estimation with aggregated data. *Journal of Econometrics* 22, 317–322. [MR714101 (85b:62059).]

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[47] Jerzy Baksalary, Tadeusz Caliński, Radosław Kala (1983). Estymacja krzywych wzrostu w układzie bloków kompletnych. *Biuletyn Oceny Odmian / Cultivar Testing Bulletin (Poznań)* 10, 105–117. [Article in Polish. English translation of article title: "Estimation of growth curves in a complete block design".]

[48] Jerzy K. Baksalary, Radosław Kala (1983). Partial orderings between matrices one of which is of rank one. *Bulletin of the Polish Academy of Sciences: Mathematics* 31, 5–7. [MR717117 (85j:15016, E. Deutsch); Zbl 0535.15006.]

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[51] J. K. Baksalary, R. Kala (1983). On the distribution of a nonnegative difference between two $\chi^2$-distributed second degree polynomial statistics. *Zastosowania Matematyki / Applicationes Mathematicae (Warsaw)* 18, 55–59. [MR714320 (85e:62100, Mir Maswood Ali); Zbl 0526.62046.]

[52] J. K. Baksalary, R. Kala, K. Kłaczyński (1983). The matrix inequality $\mathrm{M} \geq \mathrm{B}^*\mathrm{MB}$. *Linear Algebra and its Applications* 54, 77–86. [MR714874 (85h:15022, I. Olkin); Zbl 0516.15010.]

[53] Jerzy K. Baksalary (1984). Comparing stochastically restricted estimators in a linear regression model. *Biometrical Journal / Biometrische Zeitschrift* 26, 555–557. [MR773202 (86e:62041, J. Kleffe).]

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[55] Jerzy K. Baksalary (1984). A study of the equivalence between a Gauss–Markoff model and its augmentation by nuisance parameters. *Mathematische Operationsforschung und Statistik, Series Statistics* 15, 3–35. [MR729609 (85h:62085, Robert L. Mason); Zbl 0556.62045 (E. W. Grafarend).]

[56] Jerzy K. Baksalary, Jan Hauke (1984). Inheriting independence and chi-squaredness under certain matrix orderings. *Statistics & Probability Letters* 2, 35–38. [MR729289 (86a:62082, Dennis L. Young); Zbl 0584.62075 (D. N. Shanbhag).]

[57] Jerzy K. Baksalary, Anna Molińska (1984). Nonnegative unbiased estimability of linear combinations of two variance components. *Journal of Statistical Planning and Inference* 10, 1–8. [MR752448 (86b:62106, Júlia Volaufová); Zbl 0563.62045.]

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[59] Jerzy K. Baksalary (1985). Strong unified-least-squares matrices for a general linear model. *Linear Algebra and its Applications* 70, 61–65. [MR808531 (87a:62080, I. Olkin); Zbl 0584.62076.]

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[150] Jerzy K. Baksalary, Oskar Maria Baksalary (2003). Solution 30-7.1 (to Problem 30-7: “A condition for an idempotent matrix to be Hermitian” proposed by Götz Trenkler). *Image: The Bulletin of the International Linear Algebra Society* 31, 41.

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[156] Jerzy K. Baksalary, Jan Hauke (2003). Solution 29-9.1 (to Problem 29-9: "Equality of two nonnegative definite matrices" proposed by Yongge Tian). *Image: The Bulletin of the International Linear Algebra Society* 30, 29–30.

[157] Jerzy K. Baksalary, Roger A. Horn (2003). Solution 29-7.1 (to Problem 29-7: "Complementary principal submatrices and their eigenvalues" proposed by Chi-Kwong Li). *Image: The Bulletin of the International Linear Algebra Society* 30, 26–27.

[158] Jerzy K. Baksalary, Xiaoji Liu (2003). Solution 29-8.1 (to Problem 29-8: "A range equality involving an idempotent matrix" proposed by Yongge Tian). *Image: The Bulletin of the International Linear Algebra Society* 30, 27.

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[160] Jerzy K. Baksalary (2004). Solution 31-3.1 (to Problem 31-3: "A range equality for block matrices" proposed by Yongge Tian). *Image: The Bulletin of the International Linear Algebra Society* 32, 23–24.

[161] Jerzy K. Baksalary (2004). An elementary development of the equation characterizing best linear unbiased estimators. *Linear Algebra and its Applications* 388, 3–6. [MR2077843 (2005f:62107, Lutz Edler); Zbl 1052.62062.]

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[163] Jerzy K. Baksalary, Oskar Maria Baksalary (2004). Solution 31-7.1 (to Problem 31-7: "On the product of orthogonal projectors" proposed by Götz Trenkler). *Image: The Bulletin of the International Linear Algebra Society* 32, 30–31.

[164] Jerzy K. Baksalary, Oskar Maria Baksalary (2004). On linear combinations of generalized projectors. *Linear Algebra and its Applications* 388, 17–24. [MR2077845; Zbl 02105727.]

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[171] Jerzy K. Baksalary, Jan Hauke (2004). Characterizations of minus and star orders between the squares of Hermitian matrices. *Linear Algebra and its Applications* 388, 53–59. [MR2077849; Zbl 02105731 (Juan Rámon Torregrosa Sánchez).]

[172] Jerzy K. Baksalary, Jan Hauke, Xiaoji Liu, Sanyang Liu (2004). Relationships between partial orders of matrices and their powers. *Linear Algebra and its Applications* 379, 277–287. [MR2039743 (2005b:15039); Zbl 1044.15011 (Juan Rámon Torregrosa Sánchez).]

[173] Jerzy K. Baksalary, Paulina Kik, Augustyn Markiewicz (2004). Solution 31-6.1 (to Problem 31-6: "A full rank factorization of a skew-symmetric matrix" proposed by Götz Trenkler). *Image: The Bulletin of the International Linear Algebra Society* 32, 27–28.

[174] Jerzy K. Baksalary, Anna Kuba (2004). Solution 31-7.2 (to Problem 31-7: "On the product of orthogonal projectors" proposed by Götz Trenkler). *Image: The Bulletin of the International Linear Algebra Society* 32, 31–34.

[175] Jerzy K. Baksalary, Xiaoji Liu (2004). An alternative characterization of generalized projectors. *Linear Algebra and its Applications* 388, 61–65. [MR2077850; Zbl 02105732.]

[176] Jerzy K. Baksalary, Oskar Maria Baksalary (2005). When is a linear combination of two idempotent matrices the group involutory matrix? Submitted for publication in *Applied Mathematics and Computation*.

[177] Jerzy K. Baksalary, Oskar Maria Baksalary (2005). An invariance property related to the reverse order law. Submitted for publication in *Linear Algebra and its Applications*.

[178] Jerzy K. Baksalary, Oskar Maria Baksalary (2005). Some particular formulae for the Moore–Penrose inverse of columnwise partitioned matrices. Paper in preparation [based on talk by Oskar Maria Baksalary at the 12th International Linear Algebra Society Conference (Regina, Saskatchewan, Canada, June 2005)].

[179] Jerzy K. Baksalary, Oskar Maria Baksalary, Katarzyna Chylińska, George P. H. Styan (2005). A specific form of the generalized inverse of a partitioned matrix useful in econometrics. Paper in preparation [based on talk by Chylińska at the 13th International Workshop on Matrices and Statistics, in celebration of Ingram Olkin's 80th birthday (Będlewo, Poland, August 2004)].

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TABLE 2.2: The 32 research journals in which 127 research papers by Jerzy K. Baksalary are published.

| | |
|---|---|
| *Aequationes Mathematicae* | 1 |
| *The American Statistician* | 1 |
| *Annales Societatis Mathematicae Polonae, Series I: Commentationes Mathematicae* | 1 |
| *The Annals of Statistics* | 5 |
| *Annals of the Institute of Statistical Mathematics (Tokyo)* | 1 |
| *Atti della Accademia Nazionale dei Lincei, Rendiconti della Classe di Scienze Fisiche, Matematiche e Naturali: Serie VIII (Rome)* | 1 |
| *Biometrical Journal / Biometrische Zeitschrift* | 3 |
| *Biometrika* | 1 |
| *Biuletyn Oceny Odmian / Cultivar Testing Bulletin (Poznań)* | 2 |
| *Bulletin de l'Académie Polonaise des Sciences, Série des Sciences Mathématiques* | 1 |
| *Bulletin of the Polish Academy of Sciences: Mathematics* | 1 |
| *The Canadian Journal of Statistics* | 4 |
| *Communications in Statistics–Theory and Methods* | 1 |
| *Computational Statistics and Data Analysis* | 1 |
| *IEEE Transactions on Automatic Control* | 1 |
| *Journal of Econometrics* | 1 |
| *Journal of Multivariate Analysis* | 1 |
| *Journal of Statistical Computation and Simulation* | 1 |
| *Journal of Statistical Planning and Inference* | 23 |
| *Linear Algebra and its Applications* | 45 |
| *Linear and Multilinear Algebra* | 2 |
| *Mathematische Operationsforschung und Statistik* | 1 |
| *Mathematische Operationsforschung und Statistik, Series Statistics* | 4 |
| *Roczniki Akademii Rolniczej w Poznaniu: Algorytmy Biometryczne i Statystyczne (Annals of the Agricultural University of Poznań: Biometrical and Statistical Algorithms)* | 9 |
| *Roczniki Polskiego Towarzystwa Matematycznego, Seria III: Matematyka Stosowana (Annals of the Polish Mathematical Society, Series III: Applied Mathematics, Warsaw)* | 2 |
| *Sankhyā, The Indian Journal of Statistics: Series A* | 5 |
| *Scandinavian Journal of Statistics* | 1 |
| *SIAM Journal on Applied Mathematics* | 2 |
| *Statistica (Bologna)* | 1 |
| *Statistics & Probability Letters* | 1 |
| *Zastosowania Matematyki / Applicationes Mathematicae (Warsaw)* | 2 |
| *Žurnal Vyčislitel′noĭ Matematiki i Matematičeskoĭ Fiziki* | 1 |

TABLE 2.3: The 43 coauthors.

| | |
|---|---|
| 38 | Oskar Maria Baksalary |
| 2 | Tadeusz Caliński |
| 1 | Katarzyna Chylińska |
| 1 | L. C. A. Corsten |
| 12 | Anita Dobek |
| 1 | Adrian C. van Eijnsbergen* |
| 1 | R. William Farebrother* |
| 2 | Stanisław Gnot |
| 1 | Jürgen Groß |
| 11 | Jan Hauke |
| 1 | Roger A. Horn* |
| 1 | Sanpei Kageyama |
| 50 | Radosław Kala |
| 1 | Krystyna Katulska |
| 2 | Paulina Kik |
| 1 | Krzysztof Kłaczyński |
| 1 | Anna Kuba* |
| 1 | Erkki Liski |
| 1 | Sanyang Liu |
| 9 | Xiaoji Liu |
| 9 | Augustyn Markiewicz |
| 3 | Thomas Mathew* |
| 1 | Sujit Kumar Mitra* |
| 1 | Anna Molińska* |
| 1 | Kenneth Nordström |
| 1 | Markku Nurhonen |
| 1 | Halim Özdemir |
| 6 | Paweł R. Pordzik |
| 4 | Friedrich Pukelsheim |
| 1 | Tarmo Pukkila* |
| 11 | Simo Puntanen |
| 3 | P. D. Puri* |
| 2 | C. Radhakrishna Rao |
| 1 | Bernhard Schipp |
| 1 | Peter Šemrl |
| 1 | Kirti R. Shah* |
| 1 | Idzi Siatkowski* |
| 12 | George P. H. Styan |
| 2 | Tomasz Szulc |
| 3 | Zenon Tabis* |
| 1 | William F. Trench |
| 7 | Götz Trenkler |
| 1 | Haruo Yanai |

** Publication(s) with no other coauthor(s).*

TABLE 2.4: The 11 coauthors each with more than 3 joint publications.

| | |
|---|---|
| 50 | Radosław Kala |
| 38 | Oskar Maria Baksalary |
| 12 | Anita Dobek |
| 12 | George P. H. Styan |
| 11 | Jan Hauke |
| 11 | Simo Puntanen |
| 9 | Xiaoji Liu |
| 9 | Augustyn Markiewicz |
| 7 | Götz Trenkler |
| 6 | Paweł R. Pordzik |
| 4 | Friedrich Pukelsheim |

TABLE 2.5: The 11 eigenvalues greater than 3 of the $\mathbf{A}'\mathbf{A}$ matrix. The coauthor associated with each eigenvalue is identified by the largest entry (in absolute value) of the associated eigenvector.

| | |
|---|---|
| 53.10 | Radosław Kala |
| 39.60 | Oskar Maria Baksalary |
| 13.94 | George P. H. Styan |
| 11.40 | Jan Hauke |
| 9.67 | Simo Puntanen |
| 9.65 | Augustyn Markiewicz |
| 9.21 | Anita Dobek |
| 7.88 | Götz Trenkler |
| 7.45 | Xiaoji Liu |
| 5.48 | Paweł R. Pordzik |
| 3.88 | Friedrich Pukelsheim |

TABLE 2.6: PhD dissertations supervised by Jerzy K. Baksalary.

Paweł R. Pordzik, Adam Mickiewicz University, Poznań, 1985. PhD dissertation in Polish: Testymatory funkcji parametrycznych w modelach liniowych. [English translation of title: "Testimators of parametric functions in linear models".]

Zenon Tabis, Adam Mickiewicz University, Poznań, 1985. PhD dissertation in Polish: Odporność i minimalność modeli liniowych ze względu na estymowalność funkcji parametryczych. [English translation of title: "Robustness and minimality of linear models with respect to estimability of parametric functions".]

Augustyn Markiewicz, Adam Mickiewicz University, Poznań, 1988. PhD dissertation in Polish: Dopuszczalne estymatory liniowe w modelach liniowych. [English translation of title: "Admissible linear estimators in linear models".]

Idzi Siatkowski, Adam Mickiewicz University, Poznań, 1990. PhD dissertation in Polish: Modele liniowe z dwiema grupami parametrów wtrąconych. [English translation of title: "Linear models with two groups of nuisance parameters".]

TABLE 2.7: Reviews by Jerzy K. Baksalary published in *Mathematical Reviews*.

MR437554 (55 #10478) Frank J. Hall, Carl D. Meyer, Jr., (1975). Generalized inverses of the fundamental bordered matrix used in linear estimation. *Sankhyā, The Indian Journal of Statistics: Series A* 37, 428–438.

MR442001 (56 #390) George A. Milliken, Fikri Akdeniz (1977). A theorem on the difference of the generalized inverses of two nonnegative matrices. *Communications in Statistics–A, Theory and Methods* 6, 73–79.

MR448720 (56 #7025) A. Bartkowiakowa (1976). Algorithms for regression analysis. *Roczniki Polskiego Towarzystwa Matematycznego, Seria III: Matematyka Stosowana* 7, 101–115.

MR448733 (56 #7038) Wiktor Oktaba (1976). Matrix relations in the analysis of variance. *Roczniki Polskiego Towarzystwa Matematycznego, Seria III: Matematyka Stosowana* 8, 81–88.

MR455227 (56 #13466) Roman Zmyślony (1976). Quadratically admissible estimators in random models. *Roczniki Polskiego Towarzystwa Matematycznego, Seria III: Matematyka Stosowana* 7, 117–122.

MR458732 (56 #16932) Stanisław Gnot, Witold Klonecki, Roman Zmyślony (1977). Uniformly minimum variance unbiased estimation in various classes of estimators, I. *Mathematische Operationsforschung und Statistik, Series Statistics* 8, 199–210.

MR468046 (57 #7885) Johan Fellman (1976). On the effect of "nuisance" parameters in linear models. *Sankhyā, The Indian Journal of Statistics: Series A* 38, 197–200.

MR480556 (58 #715) Ching Hsiang Hung, Thomas L. Markham (1977). The Moore–Penrose inverse of a sum of matrices. *Journal of the Australian Mathematical Society, Series A* 24, 385–392.

MR514647 (80h:62055) David W. Smith, R. R. Hocking, (1978). Maximum likelihood analysis of the mixed model: the balanced case. *Communications in Statistics–A, Theory and Methods* 7, 1253–1266.

MR518661 (81b:62013) Roman Różański (1978). $G_{1,-1}$-minimax estimation of the parameters of a distribution of exponential type. *Roczniki Polskiego Towarzystwa Matematycznego, Seria III: Matematyka Stosowana* 13, 59–66.

MR531457 (80e:15002) Michał Kolupa (1978/1979). Construction of an interval which contains all components of the solution vector of an inhomogeneous Cramer system of linear equations. *Przegląd Statystyczny* 25, 295–299.

MR544565 (82g:62088) I. S. Alalouf, G. P. H. Styan (1979). Estimability and testability in restricted linear models. *Mathematische Operationsforschung und Statistik, Series Statistics* 10, 189–201.

MR567938 (82e:62097) R. W. Farebrother (1979). Estimation with aggregated data. *Journal of*

*Econometrics* 10, 43–55.
MR636025 (84a:15021) M. Barel, A. Tamir (1981). Nested matrices and the existence of least majorized elements. *Linear Algebra and its Applications* 38, 65–72.
MR636034 (84a:15017) Franklin T. Luk (1981). The communality problem for Stieltjes matrices. *Linear Algebra and its Applications* 38, 161–169.
MR645257 (83h:62109) C. G. Khatri, D. Raghavarao, R. Mercado (1981). On the estimation of fixed effects in a mixed model. *Gujarat Statistical Review* 8 (1), 1–6.
MR745226 (86j:62154) D. Pfeffermann (1984). On extensions of the Gauss–Markov theorem to the case of stochastic regression coefficients. *Journal of the Royal Statistical Society, Series B* 46, 139–148.

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## 3 On some of Jerzy Baksalary's contributions to the theory of block designs

*by* Tadeusz CALIŃSKI, *Agricultural University of Poznań, Poznań, Poland*

We present a review[3] of some results obtained by Jerzy Baksalary with regard to the theory of block designs. Particular attention is drawn to his results concerning various concepts of balance, some methods of constructing block designs, the connectedness of PBIB designs, conditions for a kind of robustness of block designs, and certain criteria concerning Fisher's condition for block designs. The importance of his results is stressed. References to other relevant works in this field are also made. There is no doubt that Baksalary's contributions to experimental design are important both from a theoretical and a practical point of view.

Jerzy Baksalary became interested in the theory of block designs in the late seventies, when the Poznań school of mathematical statistics and biometry was already quite advanced in this field. He was trying to investigate the mathematical background of the various concepts related to the theory of experimental designs, particularly of block designs, a subject of intensive study in Poznań at that time.
It will be helpful first to recall that any block design can be described by its $v \times b$ incidence matrix $\boldsymbol{N} = [n_{ij}]$, with a row for each treatment and a column for each block, where $n_{ij}$ is the number of experimental units in the $j$th block receiving the $i$th treatment $(i = 1, 2, ..., v;\ j = 1, 2, ..., b)$. This matrix, together with the vector of block sizes, $\boldsymbol{k} = [k_1, k_2, ..., k_b]' = \boldsymbol{N}'\mathbf{1}_v$, the vector of treatment replications, $\boldsymbol{r} = [r_1, r_2, ..., r_v]' = \boldsymbol{N}\mathbf{1}_b$, and the total number of units ($n = \mathbf{1}_b'\boldsymbol{k} = \mathbf{1}_v'\boldsymbol{r} = \mathbf{1}_v'\boldsymbol{N}\mathbf{1}_b$, where $\mathbf{1}_a$ is an $a \times 1$ vector of ones) is used in defining various matrices that help us to understand the statistical properties of the design. In particular, an important role in studying these properties is played by the $v \times v$ matrix

$$\boldsymbol{C} = \boldsymbol{r}^{\delta} - \boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}',$$

where $\boldsymbol{r}^{\delta} = \mathrm{diag}[r_1, r_2, ..., r_v]$, $\boldsymbol{k}^{\delta} = \mathrm{diag}[k_1, k_2, ..., k_b]$ and $\boldsymbol{k}^{-\delta} = (\boldsymbol{k}^{\delta})^{-1}$. On it, the so-called intra-block analysis of the experimental data is based [see Caliński and Kageyama (2000, Section 3.2.1)]. The interest of Baksalary was at that time confined to this type of analysis.

[3] An earlier version of this article is in the booklet for the "Session on the occasion of the 60th birthday of Jerzy K. Baksalary" held at the Mathematical Research & Conference Center, Polish Academy of Sciences, Będlewo (near Poznań), Poland, 17 August 2004.

## 3.1 Concepts of balance

In one of his earliest papers in this field [Baksalary, Dobek, and Kala (1980a)], the concept of balance of a block design is considered. Two notions of balance are defined there, for connected and disconnected block designs. But first it is noted that the rank of $\boldsymbol{C}$ is strictly related to the concept of connectedness.

DEFINITION 1 (1980a). A block design is said to be connected if rank$(\boldsymbol{C}) = v - 1,$ and is said to be disconnected of degree $g - 1$, $g \geq 2$, if rank$(\boldsymbol{C}) = v - g.$

DEFINITION 2 (1980a). A connected (disconnected of degree $g - 1$) block design is said to be $V$-balanced if all the nonzero eigenvalues of its matrix $\boldsymbol{C}$, $v - 1$ $(v - g)$ in number, are equal.

DEFINITION 3 (1980a). A connected (disconnected of degree $g - 1$) block design is said to be $J$-balanced if all the nonzero eigenvalues of its matrix $\boldsymbol{C}$ with respect to the matrix $\boldsymbol{r}^{\delta}$, $v - 1$ $(v - g)$ in number, are equal.

The notion of $V$-balance can be traced back to Vartak (1963). Now, it is more commonly termed "variance-balance (VB)" [see, e.g., Raghavarao (1971, p. 54)]. The notion of $J$-balance goes back to the concept of balance introduced by Jones (1959), though implicitly already used by Nair and Rao (1948). Graf-Jaccottet (1977) introduced the term $J$-balanced, or "balanced in the Jones sense". More frequently, this type of balance is called "efficiency-balance (EB)", due to Williams (1975) and Puri and Nigam (1975a, 1975b). However, it can be shown that the introduction of the terms VB and EB has been to some extent arbitrary [see, e.g., Caliński and Kageyama (2000, Section 4.1)]. An extreme case of $J$-balance is the orthogonality of a block design.

DEFINITION 4 (1980a). A connected (disconnected of degree $g - 1$) block design is said to be orthogonal if all the nonzero eigenvalues of its matrix $\boldsymbol{C}$ with respect to the matrix $\boldsymbol{r}^{\delta}$, $v - 1$ $(v - g)$ in number, are equal to 1.

See also Corollary 2.3.3 and Remark 2.4.2 in Caliński and Kageyama (2000).

An equivalent condition is given in the following theorem.

THEOREM 2 (1980a). *If a block design is orthogonal, then the rank of its incidence matrix $\boldsymbol{N}$ is equal to 1 when the design is connected, and is equal to $g$ when the design is disconnected of degree $g - 1$.*

## 3.2 Constructional methods

Other characterizations of EB and VB designs are given in Baksalary, Dobek, and Kala (1980b), as follows.

LEMMA 1 (1980b). *A block design is connected and EB if and only if, for some positive scalar $p$, $\boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}' - p\boldsymbol{r}\boldsymbol{r}'$ is a diagonal matrix. If this is the case, the efficiency factor of the design equals $\varepsilon = np$.*

LEMMA 2 (1980b). *A block design is connected and VB if and only if, for some positive scalar $q$, $\boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}' - q\boldsymbol{1}_v\boldsymbol{1}_v'$ is a diagonal matrix.*

It may be noted that this way of defining balance is related to the early definitions based on the off-diagonal elements of the matrix $\boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}'$, called "weighted concurrences" by Pearce (1976). Thus, Lemma 2 (1980b) is equivalent to the concept of total balance (Type

$T_0$) introduced by Pearce (1976, Section 4.A) for the case when the weighted concurrences are all equal. On the other hand, Lemma 1 (1980b) is equivalent to the concept of total balance in the sense of Jones (1959), introduced for the case when the weighted concurrences are equally proportional to the products of the relevant treatment replications [see Definitions 2.4.3 and 2.4.5 in Caliński and Kageyama (2000)].

Using these characterizations of balance, Baksalary et al. (1980b) gave several theorems useful for constructing connected EB designs (Theorems 1, 4, and 5) and connected VB designs (Theorems 2 and 3). Of particular interest is a corollary following from their Theorem 4, which can be written as follows.

COROLLARY (1980b). *If $\boldsymbol{N}_h$, $h = 1, 2, ..., a$, are the incidence matrices of connected EB designs with a common number of treatments and with the replications of treatments mutually proportional among the designs, then their juxtaposition (assemblage) $[\boldsymbol{N}_1 : \boldsymbol{N}_2 : \cdots : \boldsymbol{N}_a]$ is the incidence matrix of a connected EB design, with its efficiency factor equal to the weighted average of the efficiency factors of the initial designs.*

For some applications of this result, see, e.g., Caliński and Kageyama (2003, Section 8.2.2). Further characterizations of connected designs as well as some constructions of these designs are considered in another of Baksalary's papers [Baksalary and Tabis (1985)]. In particular, of interest is the following result.

LEMMA 2 (1985). *A block design is connected if and only if it is not isomorphic, with respect to permutations of blocks and/or treatments, to a design with the incidence matrix of the form* diag$[\boldsymbol{N}_1 : \boldsymbol{N}_2 : \cdots : \boldsymbol{N}_g]$, *where $2 \leq g \leq v$ and $\boldsymbol{N}_\ell$, $\ell = 1, 2, ..., g$, are all incidence matrices of connected block designs.*

This result rephrases Theorem 3.1 of Eccleston and Hedayat (1974). Evidently, if the design is not connected (in the above sense), it is disconnected of degree $g - 1$ [see Definition 2.2.6a in Caliński and Kageyama (2000)].

From both the theoretical and practical points of view, connectedness is a desirable property of a block design. In fact, the most frequent block designs used in practice are binary (i.e., with $n_{ij} = 0$ or $n_{ij} = 1$ for every $i = 1, 2, ..., v$ and $j = 1, 2, ..., b$) and connected designs.

When designing an experiment, the research project and the experimental material available determine the treatment replications and the block sizes, i.e., the vectors $\boldsymbol{r}$ and $\boldsymbol{k}$, of a block design to be used. In Baksalary and Tabis (1985), three theorems are proved that allow one to construct binary and connected block designs for given $\boldsymbol{r}$ and $\boldsymbol{k}$, starting from a known binary block design, not necessarily connected. The first two theorems show that although disconnected designs are not desirable in general, under certain conditions they can be transformed into connected binary block designs with desired treatment replications and block sizes. The third theorem provides a sequential procedure for transforming a connected binary block design with the minimal number of experimental units into a connected binary block design with desired vectors $\boldsymbol{r}$ and $\boldsymbol{k}$, preserving in each step the property of connectedness.

## 3.3 Connectedness of PBIB designs

Another paper written by Baksalary and Tabis (1987a) concerns the connectedness of partially balanced incomplete block (PBIB) designs. These binary designs are often used when balanced incomplete block (BIB) designs with required treatment replications and block sizes are not available. The properties of a PBIB design are determined by a relevant so-called association scheme with $m$ classes [see, e.g., Raghavarao (1971, Chapter

8); Caliński and Kageyama (2003, Section 6.0.2)]. Usually, the association schemes provide connected PBIB designs, but there may be cases where the connectedness is not preserved. In this paper a theorem is proved which gives a suitable criterion for examining the connectedness of PBIB designs based on various association schemes. Its applicabilty is shown in the context of the group-divisible $m$-associate-class PBIB designs introduced by Roy (1953–1954).

In such a design there are $v = s_1 s_2 \cdots s_m$ treatments, each denoted by $m$ indices $(i_1, i_2, ..., i_m)$, where $i_1 = 1, 2, ..., s_1,\ i_2 = 1, 2, ..., s_2,\ ...,\ i_m = 1, 2, ..., s_m$. Two treatments $(i_1, i_2, ..., i_m)$ and $(j_1, j_2, ..., j_m)$ are the $u$th associates if only their first $m - u$ indices are the same. They then occur together in exactly $\lambda_u$ blocks, this number being independent of the particular pair of $u$th associates chosen, $u = 1, 2, ..., m$. [See also Raghavarao (1971, Section 8.12.6).] In practice, PBIB designs of this type of association scheme are used mainly for $m = 2$ or $m = 3$. But the established criterion (their Corollary 1, below) can be applied for any $m$, thus extending the previously known results of Kageyama (1982) and Ogawa, Ikeda, and Kageyama (1984).

COROLLARY 1 (1987a). *A group-divisible $m$-associate-class PBIB design is connected if and only if $\lambda_m > 0$ (where $\lambda_m$ is the number of blocks in which any two treatments being the $m$th associates occur together).*

This will be illustrated by an example [Example 6.0.7 in Caliński and Kageyama (2003)]. The following incidence matrix shows a group-divisible 3-associate-class PBIB design with parameters $v = b = 8$, $r = k = 4$, $s_1 = s_2 = s_3 = 2$, $\lambda_1 = 2$, $\lambda_2 = 1$, $\lambda_3 = 2$, with the eight treatments as $(1,1,1), (1,1,2), (1,2,1), (1,2,2), (2,1,1), (2,1,2), (2,2,1), (2,2,2)$.

$$
\begin{array}{c}
(1,1,1)\\ (1,1,2)\\ (1,2,1)\\ (1,2,2)\\ (2,1,1)\\ (2,1,2)\\ (2,2,1)\\ (2,2,2)
\end{array}
\quad
\left[\begin{array}{cccc|cccc}
1 & 1 & 0 & 0 & 1 & 0 & 1 & 0\\
1 & 1 & 0 & 0 & 0 & 1 & 0 & 1\\
0 & 0 & 1 & 1 & 1 & 0 & 0 & 1\\
0 & 0 & 1 & 1 & 0 & 1 & 1 & 0\\
1 & 0 & 0 & 1 & 0 & 0 & 1 & 1\\
0 & 1 & 1 & 0 & 0 & 0 & 1 & 1\\
1 & 0 & 1 & 0 & 1 & 1 & 0 & 0\\
0 & 1 & 0 & 1 & 1 & 1 & 0 & 0
\end{array}\right]
$$

Evidently, this design (which is a 2-resolvable design) could well be used for a $2^3$ factorial experiment, which would allow the contrast between main effects of one of the factors to be estimated in the intra-block analysis with full efficiency.

## 3.4 Robustness of block designs

Another subject of interest studied by Baksalary was related to the robustness of block designs against the unavailability of data. Three sufficient conditions for a block design to be maximally robust have been derived by Baksalary and Tabis (1987b). They have used the following definition.

DEFINITION (1987b). Let a block design $\mathcal{D}$ be binary and connected, let $r_{[v]}$ denote the smallest treatment replication of $\mathcal{D}$, and let $\mathcal{D}_{\#}$ denote a design obtained from $\mathcal{D}$ by deleting any $r_{[v]} - 1$ blocks. Then $\mathcal{D}$ is said to be maximally robust against the unavailability of data and with respect to the estimability of treatment contrasts if $\mathcal{D}_{\#}$ is connected irrespective of the choice of the blocks deleted.

Their main results are as follows.

THEOREM 1 (1987b). *Let a block design $\mathcal{D}$ be binary and connected, and let $r_{[1]} \geq r_{[2]} \geq \cdots \geq r_{[v]}$ and $k_{[1]} \geq k_{[2]} \geq \cdots \geq k_{[b]}$ be its treatment replications and block sizes. Then the condition*

$$k_{[r_{[v]}]} + k_{[b]} > v$$

*is sufficient for $\mathcal{D}$ to be maximally robust against the unavailability of data and with respect to the estimability of treatment contrasts.*

THEOREM 2 (1987b). *Let a block design $\mathcal{D}$ be binary and connected, and let $r_{[1]} \geq r_{[2]} \geq \cdots \geq r_{[v]}$ and $k_{[1]} \geq k_{[2]} \geq \cdots \geq k_{[b]}$ be its treatment replications and block sizes. Further, let $\kappa_*$ and $\lambda_*$ denote the smallest off-diagonal elements of $\boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}'$ and $\boldsymbol{N}\boldsymbol{N}'$, respectively, and let*

$$K = \sum_{j=1}^{r_{[v]}-1} k_{[j]} \quad \textit{and} \quad L = \sum_{j=1}^{r_{[v]}-1} k_{[j]}^2.$$

*Then each of the conditions*

$$\kappa_* > K/[4k_{[b]}(v - k_{[b]})]$$

*and*

$$\lambda_* > L/[4k_{[b]}(v - k_{[b]})]$$

*is sufficient for $\mathcal{D}$ to be maximally robust against the unavailability of data and with respect to the estimability of treatment contrasts.*

An immediate consequence of Theorem 2 (1987b) is the following result.

COROLLARY 2 (1987b). *Let a block design $\mathcal{D}$ be binary and connected, let $r_{[1]} \geq r_{[2]} \geq \cdots \geq r_{[v]}$ and $k_{[1]} \geq k_{[2]} \geq \cdots \geq k_{[b]}$ be its treatment replications and block sizes, and let $K$ be as defined in Theorem 2 (1987b). If $\mathcal{D}$ is VB and*

$$\frac{n-b}{v(v-1)} > \frac{K}{4k_{[b]}(v-k_{[b]})},$$

*or if $\mathcal{D}$ is EB and*

$$\frac{(n-b)r_{[v-1]}r_{[v]}}{n^2 - \boldsymbol{r}'\boldsymbol{r}} > \frac{K}{4k_{[b]}(v-k_{[b]})},$$

*then $\mathcal{D}$ is maximally robust against the unavailability of data and with respect to the estimability of treatment contrasts.*

This is due to the fact that a connected and binary block design is VB if and only if

$$\boldsymbol{C} = \boldsymbol{r}^{\delta} - \boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}' = \frac{n-b}{v-1}(\boldsymbol{I}_v - v^{-1}\boldsymbol{1}_v\boldsymbol{1}_v'),$$

and is EB if and only if

$$\boldsymbol{C} = \boldsymbol{r}^{\delta} - \boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}' = \frac{n(n-b)}{n^2 - \boldsymbol{r}'\boldsymbol{r}}(\boldsymbol{r}^{\delta} - n^{-1}\boldsymbol{r}\boldsymbol{r}')$$

[see, e.g., Caliński and Kageyama (2000, Section 2.4)].

Another consequence of Theorem 2 (1987b) is the following result originally given by Ghosh (1982).

COROLLARY 3 (1987b). *Every BIB design is maximally robust against the unavailability of data and with respect to the estimability of treatment contrasts.*

Further results on this topic are given by Kageyama and Saha (1987), Kageyama (1987), Baksalary and Puri (1990), and Baksalary and Hauke (1992). For other references see Caliński and Kageyama (2003, Section 10.2).

## 3.5 Fisher's condition

Attention should also be paid to an interesting paper by Baksalary and Puri (1988) concerning Fisher's (1940) condition for BIB designs. The paper extends some earlier result obtained by Baksalary et al. (1980a) with regard to a direct relationship between EB of a block design and the rank of its incidence matrix $\boldsymbol{N}$. They have replaced the so-called Fisher's inequality, $v \leq b$, by *Fisher's condition*, defined as follows.

DEFINITION (1988). A block design is said to satisfy Fisher's condition if the rows of its incidence matrix are linearly independent.

Baksalary and Puri (1988) have obtained necessary and sufficient conditions that give complete characterizations of all combinatorially-balanced (also called pairwise-balanced) and VB designs which satisfy Fisher's condition (and, consequently, Fisher's inequality). Their main results are as follows.

THEOREM 1 (1988). *A combinatorially-balanced $($not necessarily binary$)$ block design satisfies Fisher's condition if and only if*

$$r_1^* > \lambda - \frac{\lambda}{1+\lambda\xi} \quad \textit{and} \quad r_2^* > \lambda,$$

*where $r_1^*$ and $r_2^*$, $r_1^* \leq r_2^*$, are the two smallest numbers among $r_i^*$, $i = 1, 2, ..., v$, the diagonal elements of the concurrence matrix, $\boldsymbol{N}\boldsymbol{N}'$, of the design, and where $\lambda$ is the constant off-diagonal element of that matrix and $\xi = \sum_{i=2}^{v} 1/(r_i^* - \lambda)$.* (Recall that a block design is combinatorially-balanced if the off-diagonal elements of its matrix $\boldsymbol{N}\boldsymbol{N}'$ are all equal.)

THEOREM 2 (1988). *A connected VB $($not necessarily binary$)$ block design satisfies Fisher's condition if and only if*

$$r_1 > \theta - \frac{\theta}{v+\theta\zeta} \quad \textit{and} \quad r_2 > \theta,$$

*where $r_1$ and $r_2$, $r_1 \leq r_2$, are the smallest treatment replications, and where $(v-1)\theta = n - \mathrm{tr}(\boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}')$ and $\zeta = \sum_{i=2}^{v} 1/(r_i - \theta)$.*

These results strengthen those given by Kageyama and Tsuji (1980, 1984). Certainly, they also complete the result of Baksalary et al. (1980a) for EB designs, which now may be written as follows.

THEOREM 1 (1980a). *An EB but not orthogonal block design satisfies Fisher's condition, irrespective of the connectedness or disconnectedness of the design.*

It may be mentioned here that a more general result can be stated as follows.

*A block design satisfies Fisher's condition if and only if the following two equivalent conditions hold*:
(a) *the matrix* $\boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}'$ *has no zero eigenvalues*;
(b) *the matrix* $\boldsymbol{C} = \boldsymbol{r}^{\delta} - \boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}'$ *has no unit eigenvalue with respect to* $\boldsymbol{r}^{\delta}$.

For a proof, see Corollary 2.3.1 in Caliński and Kageyama (2000). Note, finally, that the latter result corresponds to the following result given in Baksalary (1989). It can be written as follows.

COROLLARY 2 (1989). *A block design with a* $v \times b$ *incidence matrix* $\boldsymbol{N}$ *satisfies the condition*

$$\mathrm{rank}(\boldsymbol{N}) = v - \rho$$

*if and only if its matrix* $\boldsymbol{C}$ *has the unit eigenvalue with respect to* $\boldsymbol{r}^{\delta}$ *of multiplicity* $\rho$. *In particular, the design satisfies Fisher's condition if and only if all the eigenvalues of* $\boldsymbol{C}$ *with respect to* $\boldsymbol{r}^{\delta}$ *are strictly less than one, i.e.*, $\rho = 0$.

This result can also be expressed in terms of the intrablock estimation of some treatment contrasts, because the unit eigenvalue of $\boldsymbol{C}$ with respect to $\boldsymbol{r}^{\delta}$ implies that certain of these contrasts can be estimated intrablock with full efficiency. [For more on this, see Caliński and Kageyama (2000, Sections 2.3 and 3.2).]

## 3.6 Conclusions

Concluding, it can be said that several results of Baksalary, obtained usually with some co-authors, have clarified certain important aspects of the theory of block designs, particularly those related to

(a) conditions for various concepts of balance,
(b) constructional methods for EB and VB block designs,
(c) conditions for constructing desirable connected designs, PBIB designs in particular,
(d) conditions for a kind of robustness of block designs,
(e) criteria concerning the validity of Fisher's condition for block designs.

Further results of Baksalary, useful for the theory of block designs, concern the estimation of variance components under a mixed model approach, as can be seen, e.g., in Baksalary, Dobek, and Gnot (1990), or in Baksalary, Gnot, and Kageyama (1995). This line of research is, however, beyond the scope of the present paper.

It should also be mentioned that Baksalary later extended his interest from block designs to the two-way elimination of heterogeneity designs, giving further interesting results, e.g., in the papers Baksalary and Shah (1992) and Baksalary and Siatkowski (1993).

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## 4 Photographs of Jerzy Baksalary

Figure 1.

*Top left: With daughter Katarzyna and Dietrich von Rosen*
*– Poznań, May 1984 (photograph by Simo Puntanen).*
*Top right: Chicago, Illinois, May 1988 (photograph by Simo Puntanen).*
*Bottom left: With (from left to right) – Erkki Liski, Sheila Mitra (partially hidden), Sujit Kumar Mitra, Friedrich Pukelsheim, George Styan and Tarmo Pukkila:*
*– Neuchâtel, August 1989 (photograph by Simo Puntanen).*
*Bottom right: With C. Radhakrishna Rao: Hyderabad, December 2000 (photograph by Oskar Maria Baksalary).*

Figure 2.

*Tampere, August 1990. Photograph by Simo Puntanen.*


Figure 3.

*From left to right: Hans Joachim Werner, Jerzy Baksalary, George Styan, Yongge Tian, and Simo Puntanen: Halifax, Nova Scotia, 10 June 2003. Photograph by Oskar Maria Baksalary.*

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